AP Calculus AB | Homework 0212 | Name:
Dr. Paul L. Bailey | Wednesday, February 12, 2025

Write your homework *neatly*, *in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. When appropriate, *write in sentences*.

Theorem 1. (Rolle's Theorem)
Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Suppose that $f(a) = f(b) = 0$. Then there exists $c \in (a, b)$ such that $f'(c) = 0$.

Theorem 2. (Mean Value Theorem (MVT))
Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Problem 1 (Thomas §4.2 # 4)**.** Let $f(x) = \sqrt{x-1}$. Let $a = 1$ and $b = 3$. Find $c \in [a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Problem 2 (Thomas §4.2 # 10)**.** Let

$$f(x) = \begin{cases} 3 & \text{for } x = 0 \\ -x^2 + 3x + a & \text{for } x \in (0, 1) \\ mx + b & \text{for } x \in [1, 2] \end{cases}$$

For what values of a, m, and b does f satisfy the hypothesis of the Mean Value Theorem on the interval $[0, 2]$?

Problem 3 (Thomas §4.2 # 15)**.** Show that the function

$$f(x) = x^4 + 3x + 1$$

has exactly one zero on $[-2, -1]$.

Problem 4 (Thomas §4.2 # 19)**.** Show that the function

$$r(\theta) = \theta + \sin^2(\theta/3) - 8$$

has exactly one zero on $\mathbb{R}$.

Problem 5 (Thomas §3.7 # 27)**.** A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (measured in meters) increases at a steady 10 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$ m?

Problem 6 (Thomas §3.6 # 46)**.** Consider the equation

$$(x^2 + y^2)^2 = (x - y)^2.$$

Find the slope of the curve at $(1, 0)$ and $(1, -1)$.

Problem 7 (Thomas §4.1 #4)**.** Let

$$f(x) = \frac{x + 1}{x^2 + 2x + 2}.$$

Find all local extreme values of the function f, and where they occur.

Problem 8. Let

$$f(x) = x^3 - 7x + 6.$$

Let $a, b, c \in \mathbb{R}$ with $a < b < c$ and $f(a) = f(b) = f(c)$. Let $A = [a, c]$ and $B = f(A)$. Write B in interval notation.

Problem 9. Consider the polynomial

$$f(x) = x^4 - 2x^2 - 15.$$

Find all real zeros of the f. (Hint: Factor by Substitution $u = x^2$)

Problem 10. Consider the polynomial

$$f(x) = 3x^3 + 11x^2 - 19x + 5.$$

Find all real zeros of the f. (Hint: Rational Zeros Theorem)